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Impact of Energy Gain and Subsystem Characteristics on Fusion Propulsion Performance

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Rapid transport of large payloads and human crews throughout the solar system requires propulsion systems having very high specific impulse ($I_{sp} \geq 10^4$ to 10^5 s). It also calls for systems with extremely low mass-power ratios ($\alpha \leq 10^{-1}$ kg/kW). Such low α are beyond the reach of conventional power-limited propulsion, but may be attainable with fusion and other nuclear concepts that produce energy within the propellant. The magnitude of energy gain must be large enough to sustain the nuclear process while still providing a high jet power relative to the massive energy-intensive subsystems associated with these concepts. This paper evaluates the impact of energy gain and subsystem characteristics on α. Central to the analysis are general parameters that embody the essential features of any "gain-limited" propulsion power balance. Results show that the gains required to achieve $\alpha \sim 10^{-1}$ kg/kW with foreseeable technology range from ~ 100 to over 2000, which is three to five orders of magnitude greater than current fusion state of the art. Sensitivity analyses point to the parameters exerting the most influence for either 1) lowering α and improving mission performance or 2) relaxing gain requirements and reducing demands on the fusion process. The greatest impact comes from reducing mass and increasing efficiency of the thruster and subsystems downstream of the fusion process. High relative gain, through enhanced fusion processes or more efficient drivers and processors, is also desirable. There is a benefit in improving driver and subsystem characteristics upstream of the fusion process, but it diminishes at relative gains ≥ 100.

Nomenclature

A, B	=	start and end points of arbitrary straight-line trajectory
D_{AB}	=	distance between points A and B
e	=	fractional power from onboard power supply
f	=	fraction of power recirculated to driver
f_α	=	ratio of mass-power parameters
G	=	energy gain in nuclear process
G_{MIN}	=	minimum energy gain for net positive power
g	=	Earth surface gravitational acceleration
g_G	=	relative gain
h, k	=	constants in trip time expressions
I_{sp}	=	specific impulse
m	=	mass
$\dot{m}$	=	mass flow rate
m_{A2}	=	final vehicle mass after round trip return to A
m_{pay}	=	payload mass
m_{prop}	=	propellant mass
P_{in}	=	power input to propellant
P_{out}	=	power output of exhaust
R_{AB1}	=	outbound mass ratio m_{A1}/m_B
R_{BA2}	=	inbound mass ratio m_B/m_{A2}
T	=	thrust
U	=	dimensionless velocity ratio
V	=	velocity
V_e	=	exhaust velocity
X, Y, Z	=	parameter combinations in trip time expressions
α	=	overall system mass-power ratio
$\hat{\alpha}$	=	subsystem mass-power ratio
α_0	=	gain-independent portion of α
α_∞	=	gain-dependent portion of α
β	=	propellant tankage mass fraction
ΔV	=	mission velocity increment
η	=	subsystem efficiency
λ_{pay}	=	payload to vehicle inert dry mass fraction
τ	=	trip time
Φ	=	mass-power ratio factor

Subscripts

$(m)_{i,f}$	=	initial, final
$(\hat{\alpha}, \eta, m, P)_D$	=	driver
$(\hat{\alpha}, \eta, m, P)_H$	=	heat disposal
$(\hat{\alpha}, \eta, m, P)_P$	=	power processor
$(\hat{\alpha}, \eta, m, P)_S$	=	power supply
$(\hat{\alpha}, \eta, m, P)_T$	=	thruster

Introduction

THE goal of rapid interplanetary space flight will require the development of new propulsion systems based on advanced forms of nuclear energy. Over the last several decades many propulsion concepts have been studied that would enable multimonth roundtrips to Mars and missions to the furthest outer planets on the order of a year. Borowski[1] and Williams and Borowski[2] have shown that the large ΔV and vehicle accelerations required for such missions demand propulsion systems having not only very high exhaust velocities ($I_{sp} \geq 10^4$–10^5 s) but also extremely low mass-power ratios ($\alpha \leq 10^{-1}$ kg/kW).

High-energy electric propulsion systems could achieve the performance necessary for multimonth transits to Mars and near-Earth asteroids. Such power-limited concepts can also provide the I_{sp} required for extremely large-ΔV missions. However, the α may be too high to achieve the accelerations necessary for rapid excursions to the outer solar system. Faster missions on the order of a year will require systems that produce more jet power in the exhaust than that provided from onboard sources.

Such a net energy gain can be achieved in systems where nuclear reactions occur within a portion of the propellant. This energy gain is expressed as G, defined as the effective power output from the local nuclear process divided by the power required to "drive" it. The total power produced must be sufficient to provide both thrust and the power needed to drive and sustain the nuclear process, which

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can be quite significant. This type of system is "gain-limited" in that the driver can consume a large fraction of the total power produced. Nuclear fusion is the most familiar example of a gain-limited system, although some concepts involving fission also fit the definition.

Determining the upper limits of energy gain has been a central issue in fusion research over the last several decades, and it certainly plays a major part in dictating the near-term viability of fusion-based propulsion. In recent years high-energy plasma experiments have achieved local scientific gains of up to 0.5 for very short times with large, specialized facilities. For ground-based commercial power it is generally accepted that system-level engineering gains of over 50 will be required to make fusion economically competitive with other technologies. However for space propulsion there is much less consensus on what values of gain will ultimately be required to make fusion superior to other high-I_{sp} alternatives, such as electric propulsion.

In what is probably the most comprehensive assessment of fusion propulsion to date, Williams and Borowski[2] evaluated the mass properties, power output, jet power, and thrust of 13 different concepts. In addition to identifying the propulsion characteristics having the greatest impact on vehicle performance, they examined the disparity in design assumptions and parameters among the different concepts. Their conclusions consisted of several suggestions for guiding research investments and improving the fidelity of future investigations. However, the study treated the fusion process for each concept as fixed and did not evaluate how energy gain—an uncertain parameter at this early stage of development—can influence performance.

We attempt to address the issue of energy gain by investigating its effect on the overall α of fusion-propelled spacecraft. Unlike Williams and Borowski, we use a generic power balance to obtain a relationship between α, gain and general parameters that reflect subsystem mass properties and efficiencies upstream and downstream of the fusion process. Using this equation and design information from three previous fusion propulsion studies, we determine the range of gain required to meet the α requirements for ambitious interplanetary missions and assess the sensitivity of α to gain and other subsystem characteristics.

Definition of Gain-Limited Propulsion

Almost all propulsion systems in use today are classified as being either energy or power-limited. The former type, which is best represented by chemical rockets, derives all of its propulsive energy from exothermic reactions within the propellant. Total impulse and overall performance is limited by the quantity of propellant carried onboard the spacecraft. These systems typically exhibit very low exhaust velocities, but can deliver high accelerations caused by their ability to heat large amounts of propellant quickly and efficiently.

Power-limited systems, such as electric propulsion, utilize a separate onboard power source to impart energy to the propellant. Although the jet power is always less than that provided from the source, power-limited concepts can achieve high exhaust velocities and I_{sp}. Unlike chemical systems, the total impulse is limited by available power, and performance is further constrained by the lower limits of attainable α. High values of α translate into more massive systems, which yield correspondingly lower accelerations and longer trip times.

Fusion and other gain-based systems share some of the features of energy and power-limited systems, but are ultimately restricted by the net energy produced by the nuclear process, hence the term gain-limited. Because of the small burnup fraction of reactants—even at high gain $G > 100$—only a small portion of the available binding energy in the nuclear fuel is utilized, and the system is relatively independent of the total energy content of the reactants. Similarly, the total power system mass is not directly proportional to exhaust power.

As gain increases, the fraction of delivered power that must be diverted back into the driver decreases. In effect, this reduces the impact of driver mass and efficiency on the system. This is important because the driver usually has a higher mass-power ratio $\hat{\alpha}_D$ and lower efficiency η_D than the other subsystems. The performance of a gain-limited system can be improved by either reducing the $\hat{\alpha}$ of its various power-intensive subsystems or increasing G in the nuclear process.

Mission Requirements and Performance

The performance of power- and gain-limited propulsion systems is characterized by the parameters α, I_{sp}, and vehicle acceleration. These parameters are interdependent in that only two can be independently specified at once. If vehicle acceleration is treated as a dependent variable, then the objective is to understand how different combinations of α and I_{sp} affect mission performance.

We limit our consideration to performance requirements representative of fast, crewed interplanetary missions throughout the solar system. We are primarily interested in conducting round-trip missions between points A and B in as short of time as possible. Williams[3] showed that this requirement is embodied in the two parameters of trip time τ and the distance between points A and B, D_{AB}. By assuming 1) vehicle acceleration much greater than the local gravitational acceleration of the sun, 2) constant thrust, and 3) vehicle velocity of zero at A and B for both legs of the mission, one can employ a straight-line trajectory, which greatly simplifies trajectory performance calculations. That is, the vehicle thrusts at a constant I_{sp} and mass flow rate to a point approximately midway in the trajectory, whereupon it turns around and decelerates to zero velocity at the destination.

Williams[3] applied these assumptions to derive analytical approximations for τ based on D_{AB}, I_{sp}, and specific power $\sim 1/\alpha$. Borowski[1] and Kammash[4] adopted a simpler approach and obtained a single equation for τ as a function of D_{AB}, I_{sp}, and vehicle acceleration. In Appendix A we rederive a general equation for τ that is slightly different than that in Ref. 4 and eventually more useful for illustrating dependence on α:

$$\tau = D_{AB}/gI_{sp} \cdot (h + kU) \tag{1}$$

where

$$U = \frac{gI_{sp}}{\sqrt{D_{AB}(T/m_{A2})}}$$

For the outbound leg from A to B, $(h, k) = (3, 2)$, whereas for the return leg from B to A, $(h, k) = (1, 2)$. Therefore, $(h, k) = (4, 4)$ for the total round trip. The travel time is expressed as a function of distance D_{AB}, I_{sp}, and the final vehicle acceleration T/m_{A2}.

To express τ in terms of α, we derive an equation for final vehicle acceleration T/m_{A2} from a general relationship for final vehicle mass and substitute it into Eq. (1). The equation for final vehicle mass (or dry mass) is the same for both gain- and power-limited systems and contains the basic elements of payload, power-sensitive equipment αP_{out}, and propellant-sensitive structure βm_{prop}, that is,

$$m_{A2} = m_{pay} + \alpha P_{out} + \beta m_{prop} \tag{2}$$

The total power output of the propulsion system is the jet power $P_{out} = TV_e/2$, whereas the total propellant is a function of total impulse and exhaust velocity $m_{prop} = T\tau/V_e$. To simplify, we assume a constant ratio between payload and dry mass $m_{pay} = m_{A2}\lambda_{pay}$. Substituting these definitions into Eq. (2) and rearranging yields an expression for T/m_{A2}. This, in turn, is incorporated into Eq. (1) to yield the following equation for trip time:

$$\tau = 1/I_{sp}\left(X \pm \sqrt{Y + Z\alpha I_{sp}^3}\right) \tag{3}$$

where X, Y, and Z are functions independent of I_{sp} and α, that is,

$$X = \left(\frac{D_{AB}}{2g}\right)\left(2h + k^2\frac{\beta}{1-\lambda_{pay}}\right)$$

$$Y = \left(\frac{kD_{AB}}{2g}\right)^2\left(4h + k^2\frac{\beta}{1-\lambda_{pay}}\right)\frac{\beta}{1-\lambda_{pay}}$$

$$Z = k^2\frac{gD_{AB}}{1-\lambda_{pay}}$$

We neglect the negative branch of Eq. (3) because it implies a negative trip time for large I_{sp}. The placement of I_{sp} indicates that an optimum value exists, which minimizes trip time. Differentiating Eq. (3) by I_{sp} and setting to zero eventually results in a quadratic equation for I_{sp}^3 that can be solved to yield

$$I_{sp})_{opt} = (2/Z\alpha)^{\frac{1}{3}}\left[(Y + X^2) \pm X\sqrt{X^2 + 3Y}\right]^{\frac{1}{3}} \tag{4}$$

Again neglecting the negative branch, we substitute Eq. (4) into Eq. (3) to obtain minimum trip times for a specified α. This substitution also yields the interesting result that $\tau \propto \alpha^{1/3}$ and confirms that lower mass-power ratios translate to shorter trip times. Although I_{sp} is important, any system that relies on an onboard power source to provide propulsive energy will have an optimum I_{sp} that is a function of α and mission requirements.

A plot illustrating the sensitivity of round-trip time to D_{AB} and α is shown in Fig. 1, where the optimum values of I_{sp} are superimposed on lines of constant α. In assessing the requirements for interplanetary missions, we consider the parameter values necessary to complete round-trip transits between Earth and destination planets within one year.

For missions between Earth and other inner planets, which are represented by the middle band in Fig. 2, a system with $\alpha \leq 10$ kg/kW is adequate. The optimum I_{sp} values range from 3,000 to 5,000 s. Improved technology, as reflected by lower subsystem masses and α, tends to increase the optimum I_{sp} for a given destination. This is especially true for electric propulsion systems. Reducing α decreases the negative mass impact associated with the αP_{out} term in Eq. (2), thus permitting higher vehicle accelerations and shorter trip times.

The α and I_{sp} requirements become much more challenging when we consider the distances within the band representing the outer planets. The range is quite broad, extending from 4 AU to nearly 40 AU. The ambitious case of a one-year round-trip mission between Earth and Pluto (37 AU) requires an $\alpha \sim 2 \times 10^{-3}$ kg/kW and I_{sp} of

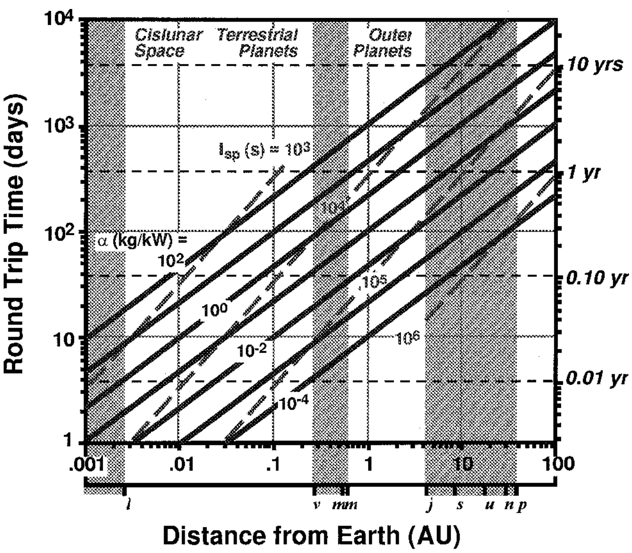

Fig. 1 Round-trip time vs distance from Earth.

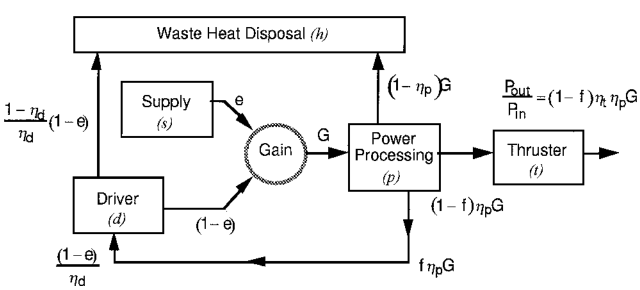

Fig. 2 Generalized power balance.

nearly 3×10^5 s. The requirements become much less severe if we consider one-year round-trip missions to Jupiter, where the required α increases to $\sim 10^{-1}$ kg/kW and the I_{sp} is $\sim$70,000 s.

These values of α are two to five orders of magnitude lower than the state of the art envisioned for nuclear electric propulsion systems. Although electric thrusters could eventually attain I_{sp} comparable to fusion concepts, it will be difficult for power system technology to meet the α requirements for rapid missions beyond the innermost outer planets, which have been determined to be $\sim 10^{-1}$ kg/kW. Ultimately, systems with α ranging from 10^{-1} kg/kW to as low as 10^{-3} kg/kW will be needed to open up the solar system to ambitious human exploration.

System Power Balance

Power- and gain-limited concepts are similar in that a large portion of their propulsion system mass can be treated as being proportional to the power delivered by the system. Thus, α is directly related to the total power balance. Both types of concepts also contain many of the same basic functions and subsystems. Therefore, it is reasonable to begin with a generic power balance that reflects the unique and common features of both.

The model used here is shown in Fig. 2. The total power needed to heat, ignite, or accelerate the propellant P_{in} is obtained from two sources—e represents the fraction of P_{in} delivered from an onboard source, whereas the remaining portion $1 - e$ comes from a driver powered by energy extracted from the heated products. The total energy from these inputs is multiplied by the factor G and then transferred to a power-processing stage, which can actually be quite complex. Most of the power, which is in the form of an energetic plasma, is passed through this stage directly to a magnetic nozzle or thruster. Only a portion of the total input power is truly "processed" to run the driver. Unusable energy arising from inefficiencies at this step and others is passed to the thermal management subsystem, which radiates waste heat to space.

An expression for α is obtained by summing the mass contributions from each of the subsystems in Fig. 2 and then dividing by the total power of the exhaust P_{out}. We assume that the mass of each subsystem can be represented as the product of a distinct mass-power ratio $\hat{\alpha}$ and the power output of that particular subsystem. Noting that the power flows in Fig. 2 are expressed as fractions of the power input to the nuclear process P_{in}, the corresponding subsystem masses are as follows

Onboard power supply:

$$m_S = \hat{\alpha}_S e P_{in}$$

Driver:

$$m_D = \hat{\alpha}_D (1 - e) P_{in}$$

Heat disposal:

$$m_H = \hat{\alpha}_H \{(1 - \eta_D)[(1 - e)/\eta_D] + (1 - \eta_D)G\} P_{in}$$

Power processor:

$$m_P = \hat{\alpha}_P \eta_P G P_{in}$$

Thruster:

$$m_T = \hat{\alpha}_T P_{out} \tag{5}$$

The output power can be expressed as $P_{out} = [(1 - f)\eta_P \eta_T G] P_{in}$. Solving for the fraction of processor power delivered to the driver yields $f = (1 - e)/(\eta_P \eta_D G)$, which upon substitution with all of the subsystem masses provides the following formulation for α:

$$\begin{aligned}\alpha = \{&\hat{\alpha}_S \eta_D e + \hat{\alpha}_D (1 - e) + \hat{\alpha}_P \eta_D \eta_P G + \hat{\alpha}_H [(1 - \eta_D)(1 - e) \\ &+ \eta_D (1 - \eta_P) G] + \hat{\alpha}_T \eta_T [\eta_P \eta_D G \\ &- (1 - e)]\} / \eta_T [\eta_P \eta_D G - (1 - e)]\end{aligned} \tag{6}$$

The effective mass-power ratio for an electric or power-limited system is obtained by setting $e = 1$ and $G = 1$ in Eq. (6) to yield

$$\alpha = \frac{\hat{\alpha}_S + \hat{\alpha}_P \eta_P + \hat{\alpha}_H (1 - \eta_P)}{\eta_T \eta_P} + \hat{\alpha}_T \tag{7}$$

The sum $\hat{\alpha}_S + \hat{\alpha}_P \eta_P$ is often treated as a single term representing the mass-power ratio of the entire power supply subsystem. Apart from the individual subsystem $\hat{\alpha}$, α is most sensitive to efficiencies of the thruster and power processing subsystems. Lower efficiencies translate into the need for a larger power source and greater capacity in the waste heat disposal subsystem. The only way to improve performance is to increase these efficiencies or reduce the mass-power ratios.

For the gain-limited case the power needed to drive the system comes from the power processing subsystem via a feedback loop. Because no major onboard power supply is required after the process has been initiated, we set $e = 0$ and retain G as a parameter in Eq. (6) to yield the mass-power ratio for a gain-limited system:

$$\alpha = \frac{\hat{\alpha}_D \eta_D + \hat{\alpha}_P \eta_D \eta_P G + \hat{\alpha}_H (1 - \eta_D) + \eta_D (1 - \eta_P) G}{\eta_T (\eta_P \eta_D G - 1)} + \hat{\alpha}_T \tag{8}$$

As before, the system mass-power ratio is a function of the subsystem $\hat{\alpha}$, including the additional parameters of driver mass and efficiency. The most unique feature is the strong dependence on gain, which is not entirely arbitrary. G must be greater than a certain minimum value in order to provide a net positive input power for driver operation. This appears as the requirement for a positive value in the denominator of Eq. (8), which can only occur when $G > G_{\mathrm{MIN}}$, where $G_{\mathrm{MIN}} = (\eta_P \eta_D)^{-1}$. At a minimum the power gained must overcome losses caused by inefficiencies in the driver and power processor subsystems. Values of G above this limit result in lower effective mass-power ratios.

In addition, α does not decrease without limit as gain is raised. Although higher gains diminish the influence of driver mass and efficiency, the power handled by the processor and thruster increases proportionally as well. Consequently, the mass influence of these subsystems, which mostly lie downstream of the fusion process, becomes dominant with increasing gain. This can be seen by taking $\lim_{G \to \infty} \alpha = \alpha_\infty$, which yields the effective lower bound for α:

$$\alpha_\infty = \hat{\alpha}_T + \frac{\hat{\alpha}_P \eta_P + \hat{\alpha}_H (1 - \eta_P)}{\eta_T \eta_P} \tag{9}$$

The parameter α_∞ primarily embodies influence of the thruster and processor. Its effect is a strong function of G and tends to diminish as $G \to G_{\mathrm{MIN}}$. As G decreases, α becomes more dependent on driver and subsystem characteristics upstream of the fusion process.

Although α approaches an infinite value at the minimum gain condition, the sensitivity to driver characteristics can be ascertained by evaluating α at $G = 0$, thereby removing the influence of gain in Eq. (8). The result α_0' is a negative quantity that reflects the impact of driver characteristics on α. This impact can be addressed more clearly by expressing α_0' as a positive value via $\alpha_0 = -\alpha_0'$, thus yielding

$$\alpha_0 = [\hat{\alpha}_D \eta_D + \hat{\alpha}_H (1 - \eta_D)]/\eta_T - \hat{\alpha}_T \tag{10}$$

Substituting α_0, α_∞, and G_{MIN} into Eq. (8) results in the following expression for α:

$$\alpha = \frac{G\alpha_\infty + G_{\mathrm{MIN}} \alpha_0}{G - G_{\mathrm{MIN}}} \tag{11}$$

Characterizing Mass-Power Ratio

Equation (11) pertains to any gain-limited propulsion concept that recirculates a portion of the output power to drive itself. Even if a power balance different than the one in Fig. 2 had been assumed, one would still arrive at the same expression in Eq. (11). Naturally, the equations for α_∞ and α_0 would be different, but they would still embody subsystem characteristics downstream and upstream of the fusion process, respectively.

Applying Eq. (11) to assess the relationship between α and G requires values of G_{MIN}, α_∞, and α_0 calculated from reasonable projections of mass properties, power flows, and efficiencies. We refer to design information from the Spherical Torus (ST), VISTA,

Table 1 Reference parameter values

Parameter	ST	VISTA	ICAN-II
Design reference gain, G	67.8	277.7	9474
Minimum gain, G_{MIN}	1.68	17.8	3.28
Infinite gain mass-power ratio, kg/kW, α_∞	0.047	0.075	0.016
Zero gain mass-power ratio, kg/kW, α_0	5.68	0.266	40.7
System mass-power ratio, kg/kW, α	0.192	0.098	0.030
Ratio of gain to minimum gain, g_G	40.4	15.6	2891
Driver to processor ratio, f_α	66.4	3.54	2540

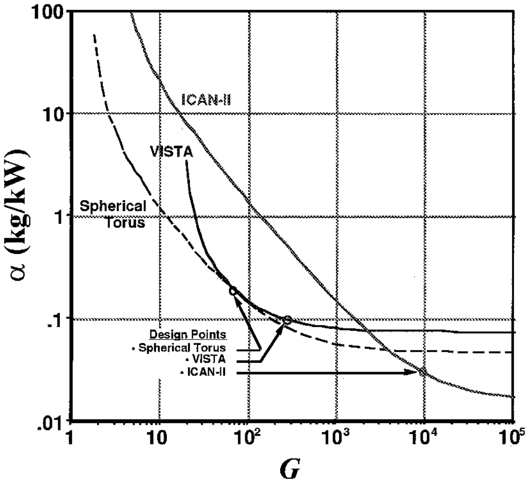

Fig. 3 Sensitivity of α to gain for different propulsion concepts.

and ICAN-II investigations[1,5–10] mainly because of the unique 1) extent of design detail, 2) thorough accounting of power flows, and 3) recognition of the significant impact of waste heat handling. Although these three concepts differ considerably, their processes are similar enough that the subsystems and power flows can be defined in accordance with Fig. 2. Resulting values of α_∞, α_0, and G_{MIN}, along with design point α and G, are listed in Table 1. These concepts are summarized further in Appendix B.

The data in Table 1 are used to construct the plots in Fig. 3, which show variation of α over an arbitrary range of G. All three plots exhibit the same general behavior. From the vertical asymptote at $G \to G_{\mathrm{MIN}}$, the values of α decrease and approach α_∞ in the limit as $G \to \infty$. In all cases the minimum mass-power ratio, represented by α_∞, lies between 0.01 and 0.1 kg/kW. However, the design point values of G yield α, which are closer to 0.1 kg/kW. The plots for ST and VISTA indicate that gains greater than 200 are required to achieve α of 0.1 kg/kW or less. For ICAN-II the required gain is over an order of magnitude higher.

A visible feature of these plots is the way their α vs G relationships differ between the limits $G \to G_{\mathrm{MIN}}$ and $G \to \infty$. For instance, most of the variation in α for VISTA takes place over a ~two-decade range of G, compared to ~four decades for ST and ~five decades for ICAN-II. In addition, ICAN-II and, to a lesser extent, ST seem to exhibit a constant exponential dependence over much of this range, while VISTA does not.

These differences arise from the competing influence of α_∞ and α_0, which is shown more effectively by expressing α in Eq. (11) as

$$\alpha = \Phi \alpha_\infty \tag{12}$$

where

$$\Phi = (g_G + f_\alpha)/(g_G - 1) \tag{13}$$

The factor Φ comprises the normalized parameters $g_G = G/G_{\mathrm{MIN}}$ and $f_\alpha = \alpha_0/\alpha_\infty$. The parameter g_G normalizes gain with respect to the minimum value required to drive the fusion process, whereas f_α reflects the relative influence between upstream and downstream subsystem characteristics. Large f_α are representative of concepts where driver mass and inefficiencies predominate, whereas small

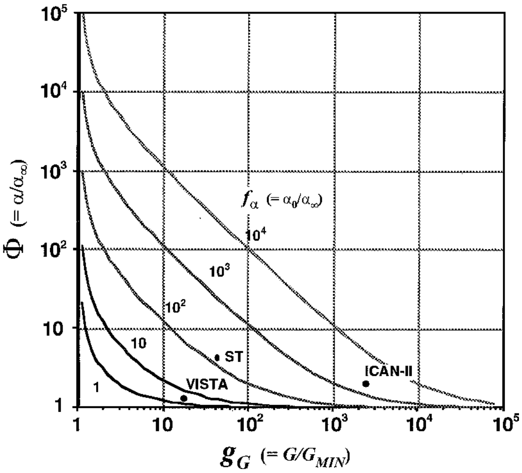

Fig. 4 Sensitivity of Φ to relative gain and subsystem characteristics.

values indicate situations where the processor and thruster subsystems exert more influence.

The recast limits of α — $\lim_{g_G \to \infty} \Phi = 1$ and $\lim_{g_G \to 1} \Phi = \infty$ — show that Φ is always greater than 1. From Eq. (13) we see that the influence of f_α is negligible at extremely high relative gains. However, its contribution becomes more significant as $g_G \to 1$. This is particularly evident in the plots of Fig. 4, which show Φ vs g_G at decade increments of f_α, along with the point-design values for VISTA, ST, and ICAN-II. Apart from the normalization of abscissa and ordinate values, these curves exhibit the same general behavior as those in Fig. 3 and are distinguished by their values of f_α.

The manner in which gain influences the system mass-power ratio is characterized by the parameter Φ. In effect, Φ represents the degree to which gain overcomes the mass penalties and inefficiencies of the driver and upstream processes. It is the factor that distinguishes gain-limited behavior from power-limited behavior. When viewed this way, gain is important only to the extent that it suppresses the negative impact of upstream processes—it has no direct bearing on α_∞.

Improving Mass-Power Ratio

Defining α in terms of Eqs. (12) and (13) is useful for illustrating the effect of gain, but does little to show which parameters have the most influence on α. Understanding the sensitivity of α to variations in g_G, f_α, and α_∞ is important because it points to the most effective way of reducing α and improving mission performance.

There are several ways in which mass-power ratio α can be reduced. One is to lower the values of α_∞ or f_α, whereas another is to increase g_G. We can determine which approach has the greatest effect by comparing the sensitivity of α to variations in these parameters. It is more realistic to evaluate these sensitivities on the basis of fractional or percent variations from the parameter values. This ensures dimensional consistency and takes into account the substantial difference in their magnitudes. For instance, a small fractional change in α_∞ is expressed as $d\bar{\alpha}_\infty = d\alpha_\infty/\alpha_\infty$, where $d\alpha_\infty$ is the differential based on the actual value of α_∞. The sensitivity of α to a fractional variation in α_∞ then becomes

$$\frac{\partial\alpha}{\partial\bar{\alpha}_\infty} = \alpha_\infty\left(\frac{\partial\alpha}{\partial\alpha_\infty}\right) \tag{14}$$

Taking the partials of α with respect to α_∞, f_α, and g_G and applying the definition in Eq. (14) yields three equations. These, in turn, are used to construct the following three comparative relations:

$$\frac{(\partial\alpha/\partial\bar{\alpha}_\infty)}{(\partial\alpha/\partial\bar{g}_G)} = -\frac{(g_G - 1)}{g_G}\left(\frac{g_G + f_\alpha}{1 + f_\alpha}\right) \tag{15}$$

$$\frac{(\partial\alpha/\partial\bar{\alpha}_\infty)}{(\partial\alpha/\partial\bar{f}_\alpha)} = \frac{g_G + f_\alpha}{f_\alpha} \tag{16}$$

$$\frac{(\partial\alpha/\partial\bar{g}_G)}{(\partial\alpha/\partial\bar{f}_\alpha)} = -\frac{(1/f_\alpha) + 1}{1 - (1/g_G)} \tag{17}$$

Equation (15) compares the sensitivity of α to percent changes in α_∞ and g_G. An absolute value greater than one means that α is more sensitive to α_∞, whereas a value less than one suggests that g_G has greater influence. The same reasoning holds for Eqs. (16) and (17), which portray similar comparisons for α_∞ vs f_α and g_G vs f_α, respectively.

We note from their definitions that $g_G > 1$ and $f_\alpha > 0$. Moreover, Table 1 shows that the lower limit of g_G is probably an order of magnitude higher —$g_G > 10$. Over the entire range of possible g_G and f_α, the absolute values of Eqs. (15) and (16) are always greater than one. This implies that the system mass-power ratio is most sensitive to changes in α_∞, especially when $g_G \gg f_\alpha$.

Although α_∞ clearly outweighs the influence of g_G and f_α, these other two parameters still have a strong effect. Applying the conditions noted before, we see that the absolute value of Eq. (17) is always greater than one. This implies that g_G has a greater influence than f_α, particularly at low values for these parameters. However when we consider the values of g_G and f_α in Table 1, their influence is almost equivalent.

α_∞, g_G, and f_α differ in an important respect: α_∞ is one of the parameters arising from the original expression for α in Eq. (8), whereas g_G and f_α are defined as ratios of these original parameters. We see that from $g_G = G/G_{\text{MIN}}$, an equivalent percentage increase in G or reduction in G_{MIN} yields the same increase in relative gain. Similarly from $f_\alpha = \alpha_0/\alpha_\infty$, the same decrease in f_α is obtained through an equivalent percent reduction in α_0 or increase in α_∞. However, intentionally increasing α_∞ to lower f_α would actually increase the value of α, in accordance with the stronger sensitivities in Eqs. (15) and (16). Thus, any effort to reduce f_α should involve changes to α_0, not α_∞.

Relaxing Gain Requirements

Rapid interplanetary space flight requires mass-power ratios equal to or lower than 0.1 kW/kg. We have shown that achieving this goal with fusion propulsion will require energy gains ranging from 200 to upwards of 2000. This range is three to five orders of magnitude greater than current fusion state of the art and certainly represents a significant technological challenge. We have already examined how variation in basic combinations of subsystem characteristics can lead to reductions in α and improved performance. However, it is equally useful to explore how these groupings can be varied in order to relax gain requirements, while still keeping α constant at a target value.

As before, we employ fractional changes to examine parameter sensitivities. A fixed value of α is assumed by setting the differential $d\alpha(\alpha_\infty, g_G, f_\alpha)$ equal to zero. This yields partials of g_G with respect to α_∞ and f_α, which can be recast as the following sensitivity equations:

$$\frac{\partial\bar{g}_G}{\partial\bar{\alpha}_\infty} = \frac{(g_G - 1)}{g_G}\left(\frac{g_G + f_\alpha}{1 + f_\alpha}\right) \tag{18}$$

$$\frac{\partial\bar{g}_G}{\partial\bar{f}_\alpha} = \frac{1 - (1/g_G)}{(1/f_\alpha) + 1} \tag{19}$$

Equations (18) and (19) show that the required gain can be decreased by lowering either α_∞ or f_α. For relative gains of two or more, the sensitivity to α_∞ is always greater than unity. This is not the case for f_α, which is always less than one. Referring to the values in Table 1, we see that a 10% reduction in α_∞ would allow us to lower the relative gain by 16–30%, whereas such a reduction in f_α would enable a decrease of only 7–10%.

Taking the ratio of Eq. (18) to Eq. (19) yields the same expression as Eq. (16) and confirms the dominant role of α_∞ compared to f_α. For the situation in which $g_G \gg f_\alpha$, the high relative gain almost completely suppresses the influence of driver and upstream characteristics. In this regime $\Phi \approx 1$, and improving driver characteristics has a negligible impact on reducing gain requirements. The stronger influence of α_∞ persists even when $g_G \ll f_\alpha$. However, in this regime, where $\Phi \gg 1$, the relative sensitivities are essentially equivalent.

Whether the interest is in improving performance or lowering gain requirements, efforts should focus on reducing the influence of subsystems downstream of the fusion process. Improving driver and upstream processes becomes relevant only at low relative gains. Otherwise, it is preferable to operate at as high of gain as possible while improving downstream subsystem characteristics.

Discussion

Our sensitivity assessments are based on parameters that represent combinations of subsystem mass-power ratios $\hat{\alpha}$ and efficiencies η appearing in the original expression for α in Eq. (8). It is reasonable to ask why the assessments were not performed using variables from this original expression. The impacts of $\hat{\alpha}$ and η could be evaluated directly without resorting to derived parameters. More equations would be necessary, but it would point to the specific $\hat{\alpha}$ and η having the most influence on α and gain.

The problem is that the primitive expression for α in Eq. (8) is a function of the power balance, which can vary from one concept to another. A sensitivity assessment based on the original expression for α would be very concept dependent and may not reflect impacts for another type of system. Our approach of using generalized parameters that apply to any power balance allows us to draw global conclusions pertaining to gain-limited performance. Nonetheless, derived parameters are one step removed from the variables characterizing spacecraft design and function. Therefore, it useful to examine the connection between these variables and the desired changes in α_∞, g_G, and f_α.

Our analysis has shown that α_∞ has the greatest impact on α. The definition of α_∞ in Eq. (9) points to several ways in which it can be reduced. The most obvious approach is to lower the mass-power ratios of the thruster, processor, and heat disposal subsystems. Another prospect is to increase thruster and processor efficiency. Thruster efficiency has a broad influence that directly affects the mass impact of the processor and heat disposal subsystems. Processor efficiency is also important, but increasing it serves primarily to neutralize the mass impact of the heat disposal subsystem.

We have identified g_G as the second most influential parameter. Increasing g_G reduces α by diminishing the influence of f_α and lowering the value of Φ in Eq. (12). g_G can be increased by either raising G or decreasing G_{MIN}. Within our context the value of G is arbitrary, but ultimately hinges on significant advancements in fusion science and technology. However, G_{MIN} can be reduced by increasing driver and processor efficiency—the latter of which is consistent with the goal of decreasing α_∞.

We have shown f_α to have the least influence of all of the parameters. The only way to reduce it, while not negatively impacting α_∞, is to lower α_0. From Eq. (10) this can be accomplished by reducing the mass-power ratios of the driver and heat disposal subsystems. Improving heat disposal characteristics is especially advantageous because it also contributes to the primary goal of lowering α_∞. Perhaps the most significant parameter in α_0 is thruster efficiency. Increasing it not only lowers α_0, but also serves to reduce α_∞.

Conclusions

Fusion propulsion offers the promise of achieving the high I_{sp} and low mass-power ratios ($\alpha \leq 10^{-1}$ kg/kW) required for rapid interplanetary space flight. In this paper we have evaluated the impact of energy gain and subsystem characteristics on α using general parameters that embody the essential features of any gain-limited propulsion concept. We have shown that, even with the optimistic technology projections from previous design studies, the required energy gains for ambitious interplanetary missions range upwards of 200 to possibly 2000 or greater. These high values are three to five orders of magnitude greater than current fusion state of the art.

Sensitivity analyses were performed to determine the influence of the general parameters α_∞, g_G, and f_α on α. Results showed that α_∞, which roughly reflects the mass-power ratios and efficiencies of subsystems downstream of the fusion process, has the greatest impact on α and sets the minimum attainable value of α. Next important are the parameters contributing to g_G, which are the absolute value of gain and driver and processor efficiency. As expected, operation at high relative gains is desirable and can be achieved via improved fusion processes or more efficient drivers and processors. Finally, improvements should focus on the terms composing α_0, which reflect processes upstream of the fusion process.

The benefits of reducing α_∞ and f_α also apply to the relaxation of gain requirements. About an assumed target value of α, reduction in α_∞ offers the greatest potential for lowering g_G. Improving driver and upstream processes are relevant only when the relative gain is low. Otherwise, it is preferable to operate at as high of gain as possible while improving downstream characteristics.

Admittedly, fusion is still at the earliest stages of development, and there is little agreement as to what values of gain will be practical or even feasible. If it appears that high values are possible ($G > 100$), then research should focus on improving the $\hat{\alpha}$ and η of the subsystems downstream of the nuclear process. This is consistent with one of the main recommendations of Williams and Borowski,[3] who pointed to magnetic nozzles as an important area for investment. On the other hand, if projections for gain are lower ($G < 100$), then research must provide dramatic improvements in $\hat{\alpha}$ and η of all subsystems, particularly the driver. This is an extremely difficult challenge and may limit the viability of fusion and gain-limited concepts in general.

Appendix A: Trip Time Derivation

The travel time at constant thrust is nothing more than the propellant mass divided by mass flow rate. Therefore, the respective transit times for the outbound and inbound legs of a continuous thrust mission can be expressed as

$$\tau_{AB} = [gI_{\mathrm{sp}}/(T/m_{A2})]R_{BA2}(R_{AB1} - 1) \tag{A1}$$

$$\tau_{BA} = [gI_{\mathrm{sp}}/(T/m_{A2})](R_{BA2} - 1) \tag{A2}$$

Because vehicle velocity is a function of time and the propellant flow rate is assumed constant $\dot{m} = T/(gI_{\mathrm{sp}})$, the distance traveled in each leg of the mission can be expressed in terms of the integral

$$D = 1/m \int_{m_i}^{m_f} V \,\mathrm{d}m$$

Substituting into this the instantaneous form of the rocket equation $V = gI_{\mathrm{sp}} \ln[m_i/m(t)]$, and integrating yields the following equations for distance traveled:

$$D_{AB} = \frac{(gI_{\mathrm{sp}})^2}{T/m_{A2}} R_{BA2}\left(\sqrt{R_{AB1}} - 1\right)^2 \tag{A3}$$

$$D_{BA} = \frac{(gI_{\mathrm{sp}})^2}{T/m_{A2}} \left(\sqrt{R_{BA2}} - 1\right)^2 \tag{A4}$$

Equations (A3) and (A4) are two distinct expressions for the mass ratios R_{AB1} and R_{BA2}. Recognizing that $D_{AB} = D_{BA}$, R_{AB1} and R_{BA2} can be solved in terms of D_{AB} and then substituted into Eqs. (A1) and (A2) to yield the generalized expression for trip time in Eq. (1).

Appendix B: Concept Summary

The ST concept, originally evaluated by Borowski[1] and studied extensively by Williams et al.,[5] employs magnetic confinement fusion (MCF) in a device geometrically similar to a tokamak. Because the MCF regime occupies the low end of plasma density, this concept requires long confinement times to sustain fusion burn and operates in a steady, continuous mode. A unique feature of the ST reactor is its low torus aspect ratio, which could mitigate many of the instabilities encountered with traditional MCF systems. Thrust is produced by drawing off a portion of heated plasma from the reactor, mixing it with injected hydrogen propellant, and directing the resulting hydrogen/fusion product plasma rearward through a magnetic nozzle. The ST concept also requires external methods of heating to raise the plasma to ignition temperatures and large magnetic fields to compress the nuclear fuel to fusion densities. The most recent assessment employs D-He3 fueled fusion to heat injected hydrogen. The vehicle is sized for a Saturn rendezvous, with a thrust and I_{sp} of 26,000 N and 38,612 s, respectively.

The VISTA concept, developed by Orth et al.,[6,7] is based on inertial confinement fusion (ICF) and represents the opposite end of the operational spectrum. A small pellet of solid fuel is compressed to ignition conditions by an intense pulse of laser energy. Although the inertia of imploding material provides confinement on timescales of only tens of nanoseconds, the densities are 10 or more orders of magnitude higher than MCF plasmas. The VISTA spacecraft is configured in the form of an inverted cone. An array of onboard lasers drives fusion in targets ejected at the cone's vertex. A magnetic field generated by a superconducting coil within the cone directs the plasma emanating from each microexplosion rearward to produce thrust. VISTA is characterized by the need for very high driver energies caused by the low efficiency of its lasers. The performance estimates in Refs. 6 and 7 are based on D-T fusion, but eventual use of less neutronic fuels, such as D-D and D-He3, is also mentioned. The VISTA studies concentrate on round-trip human missions to Mars, although the capabilities of this concept could be applied to outer solar system exploration. The thrust and I_{sp} are 185,000 N and 12,562 s, respectively.

The ICAN-II concept, conceived and investigated by researchers at Pennsylvania State University,[8–10] is a hybrid form of ICF that integrates use of particle and antimatter beams to initiate combined fission-fusion in a compressed nuclear target. A pellet of D-T fuel and U-235 is compressed with light ion beams and irradiated with a low-intensity stream of antiprotons. The antiprotons initiate a hyperneutronic fission process in the U-235 that rapidly heats and ignites the D-T core. The resulting radiation from fission and fusion is transformed via wavelength shifter material and ablates the inside surface of a hemispherical shell to produce thrust. Because the antimatter actually initiates the reactions, this approach requires lower driver energies than other ICF concepts. However, the driver power is still significant, and this approach depends on dramatic improvements in technologies for antimatter production and storage.[11] Like VISTA, the spacecraft is designed for a roundtrip mission to Mars, but with a thrust of 100,000 N and I_{sp} of 13,500 s.

In all cases the "effective" power output discounts unrecoverable losses of radiation to space in the form of high-energy photons and neutrons. This has a relatively small effect on the fusion power multiplication of ST, in which only 8.5% of the power is presumed unrecoverable. By contrast, it has a significant effect on VISTA and ICAN-II, where 72 and 84%, respectively, of the fusion power is considered lost. Other major differences among the concepts are summarized in Ref. 2.

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